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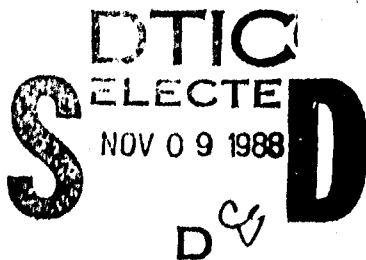


RSRE  
MEMORANDUM No. 4150

**ROYAL SIGNALS & RADAR  
ESTABLISHMENT**

**CALCULATION OF THE H-PLANE APERTURE DISTRIBUTION  
AND FAR FIELD PATTERN FOR A METAL PLATE LOADED HORN**

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AUTHOR: A H Lettington, N J Hunaban

DATE: May 1988

## SUMMARY

This note describes the results of a mathematical analysis of the aperture distribution of a horn which has been loaded with a conducting plate. The purpose of the plate is to improve the H-plane far field pattern of the horn and hence the angular resolution of a passive millimetre wave imager which uses a horn fed antenna.

The calculation is based on the Fresnel-Kirchoff diffraction formula and account is taken of multiple reflections within the horn and from the plate.

The resulting H-plane aperture distribution is far from uniform but has a sharp far field pattern. The E-plane distribution remains unchanged by the presence of the plate.



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# CALCULATION OF THE H-PLANE APERTURE DISTRIBUTION AND FAR FIELD PATTERN FOR A METAL PLATE LOADED HORN

A H LETTINGTON, N J HUNABAN

## 1. INTRODUCTION

In mmwave imaging radiometers there is a need to maintain the maximum angular resolution for a given aperture size and to ensure maximum collection efficiency. These requirements run contrary to current radar practice where the antenna aperture distribution is frequently tapered to reduce side lobes in the far field pattern. The effect of this tapering broadens the main beam width so reducing the resolution. Ideally for passive imaging the aperture distribution needs to be maintained across the full aperture or even given a negative taper towards its edges.

The aperture distribution function of an antenna is strongly influenced by the angular distribution of its feed. For a horn fed reflector this angular distribution is roughly Gaussian which in any efficient system produces a tapered aperture distribution. This effect may be minimised by relaying the image of the aperture of the feed horn directly onto the primary antenna.

This solution is ideal in the E-plane where the amplitude of the electric vector is constant across the horn aperture, but is far from ideal in the H-plane where the amplitude has a cosine dependence, dropping to zero at the horn edges.

It has been observed recently (Appleby and Lettington - to be published) that the insertion of a flat plate parallel to the axis of the horn and the E-plane can be used to improve the angular resolution in the H-plane while leaving the E-plane distribution unchanged.

The effect of this plate is particularly marked when a lens is used in the mouth of the horn to produce a flat wavefront at the exit of the lens-horn combination.

This note describes a theoretical analysis of a 2D H-plane sectorial horn with and without the inserted plate and describes the origin of the observed effect.

## 2. THE THEORY OF DIFFRACTION LIMITED WAVE PROPAGATION

The amplitude  $U(P)$  of the field at a point  $P$ , when a distant source  $P_0$  is viewed through an aperture,  $S$  is described by the well known Fresnel-Kirchoff diffraction formula (see Born and Wolf <sup>1</sup>) shown in equation 1.

$$U(P) = \frac{-iC}{2\lambda} \iint_S \frac{e^{ik(r_0 + s)}}{r_0 s} [\cos(n, r_0) - \cos(n, s)] dS \quad (1)$$

The terms in this equation are illustrated in Figure 1, which contains a point source  $P_0$ , an aperture  $S$  and a point of observation  $P$ .

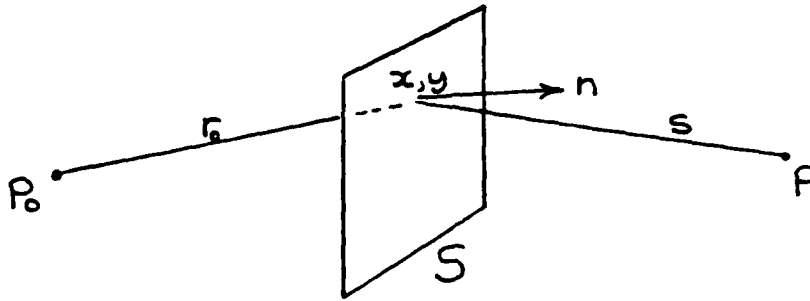


Figure 1. Illustration of the terms used in the Fresnel-Kirchoff diffraction formula (see equation 1).

The quantity  $r_0$  is the distance of the source from a particular point  $x, y$  within the aperture  $S$ , and  $s$  is the distance of this same aperture position to the point of observation  $P$ . The angles  $n, s$  and  $n, r_0$  are between the normal to the wavefront at the particular point within the aperture and the respective beams  $s$  and  $r_0$ . The wavelength of the radiation is  $\lambda$  and  $k = 2\pi/\lambda$ . The quantity  $C$  is a constant related to the strength of the distant source  $P_0$ .

Equation 1 may be rewritten as follows:-

$$U(P) = \frac{U(x, y)}{\lambda} \iint_S \frac{(1 + \cos \chi)}{2} \frac{e^{iks}}{s} dx \cdot dy \quad (2)$$

where  $U(x, y)$  is the aperture distribution function describing the field strength at an arbitrary point  $x, y$  within the aperture  $S$ . In equation 2 it has been assumed that the source is sufficiently distant from the aperture that the wavefront is plane. The quantity  $\chi$  is the angle between the normal to the wavefront in the direction of the incident

radiation and the direction of the secondary wavelet emanating from  $(x, y)$  in the direction of P.

The equation 2 may be modified for use in two dimensional calculations by changing the normalisation conditions and by performing a single instead of a double integral

$$\text{i.e. } U(P) = \frac{U(x)}{\sqrt{\lambda}} \int \frac{e^{iks}}{\sqrt{s}} \frac{(1 + \cos \chi)}{2} dx \quad (3)$$

### 3. CALCULATION OF APERTURE DISTRIBUTION FOR A 2D H-PLANE SECTORIAL HORN

The aperture distribution function for a plane horn has been calculated many times previously usually by placing a source at the phase centre in the neck of the horn. In the present case we have assumed that the wavefront at the neck of the horn is plane with a cosine aperture distribution.

Secondary wavelets then radiate from the plane wave section and arrive at a point P in the mouth of the horn. These wavelets may arrive directly or else by reflection at the walls of the horn.

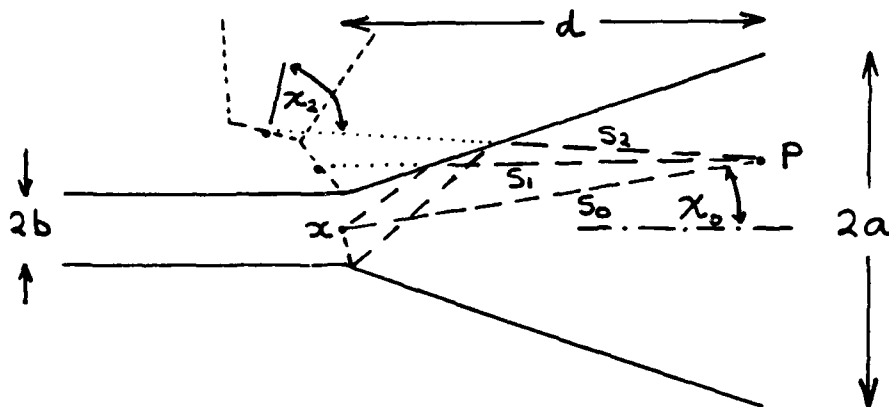


Figure 2. Illustration of a 2 dimensional sectorial horn.

The 2D sectorial horn is illustrated in Figure 2, where the paths of the single and multiply reflected wavelets are shown along with the path of the direct wavelet.

In equation 3 the origin for the integration was taken at the middle of the neck. For each point P the integration was performed over  $\pm b$ , the dimension of the waveguide, and over all possible single and multiple reflections.

The aperture distribution of the neck region was given by:

$$U(x) = \cos \left[ \frac{\pi}{2} \cdot \frac{x}{b} \right] \quad (4)$$

The quantity  $s$  in equation 3 is illustrated as  $s_0$  for the direct wavelet and as  $s_2$  for a multiply reflected path. Their respective angles  $\chi_0$  and  $\chi_2$  are similarly illustrated.

For an H-plane distribution the electric vector is out of the plane of the paper in Figure 2, and undergoes a  $\pi$  phase change at each reflection. This does not occur in reflections of E-plane radiation in the configuration shown in Figure 2.

For this reason the contributions to  $P$  from any odd numbered reflections have to be subtracted in the integration.

The complex amplitude was calculated at 21 points across the mouth of the horn from which the intensity and phase were obtained. The parameters used in this calculation were  $\lambda = 8\text{mm}$ ,  $a = 44\text{mm}$ ,  $b = 3.5\text{mm}$  and  $d = 155\text{mm}$ . The results are shown in Figure 3. It can be seen that, as expected, the intensity drops to zero at the edges of the horn due to the opposite phases of the radiation arriving directly and by a reflection at the nearest wall.

The phase distribution is typical of a spherical wavefront emitting from the mouth of the horn. The use of a lens at this point could be used to produce a plane wavefront.

#### 4. CALCULATION OF THE APERTURE DISTRIBUTION FOR A PLATE LOADED H-PLANE SECTORIAL HORN

The purpose of the plate in the horn was to invert the phase of a significant amount of the radiation arriving at the mouth of the horn after an odd number of reflections in the side wall of the horn.

The mechanical layout is illustrated in Figure 4.

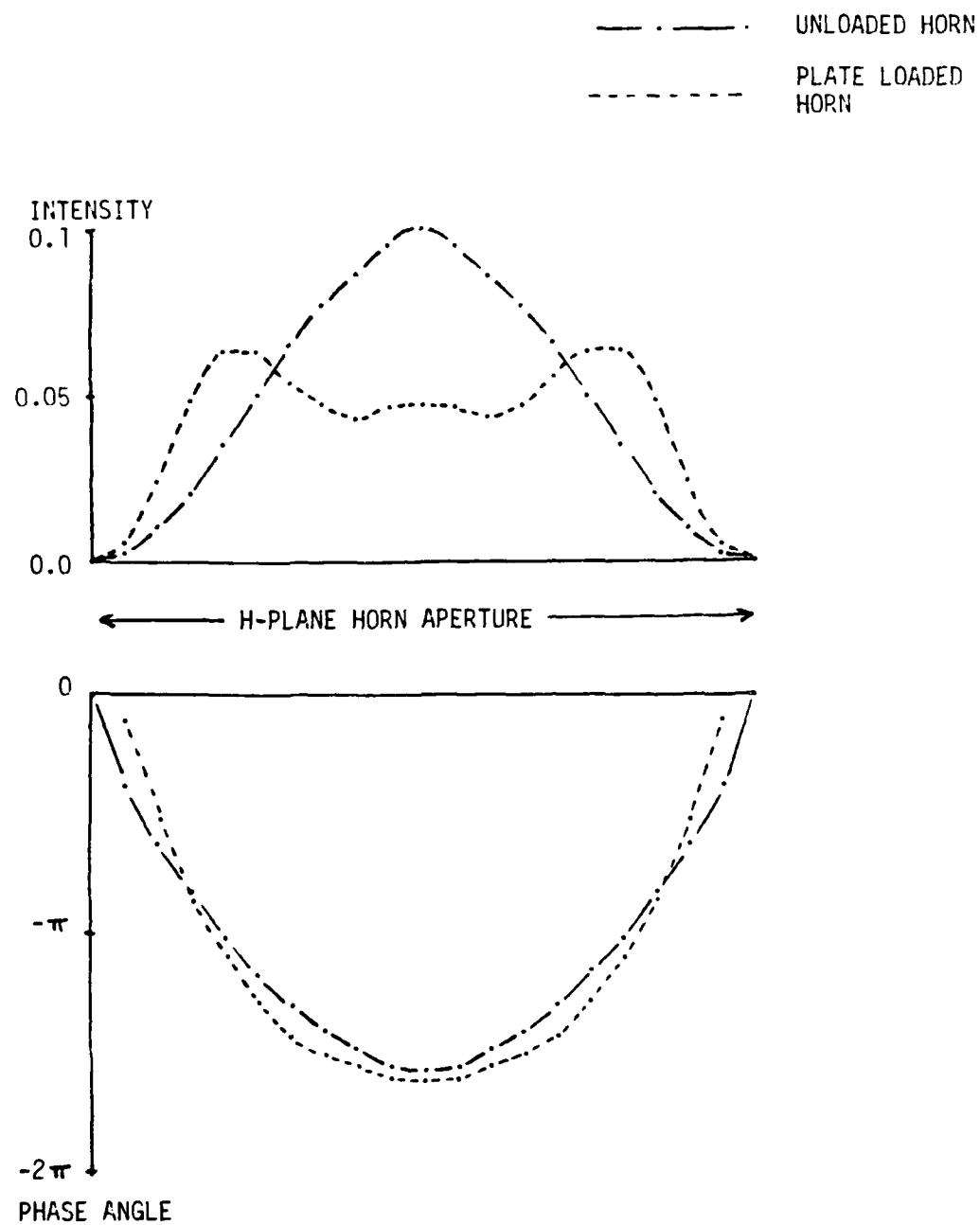
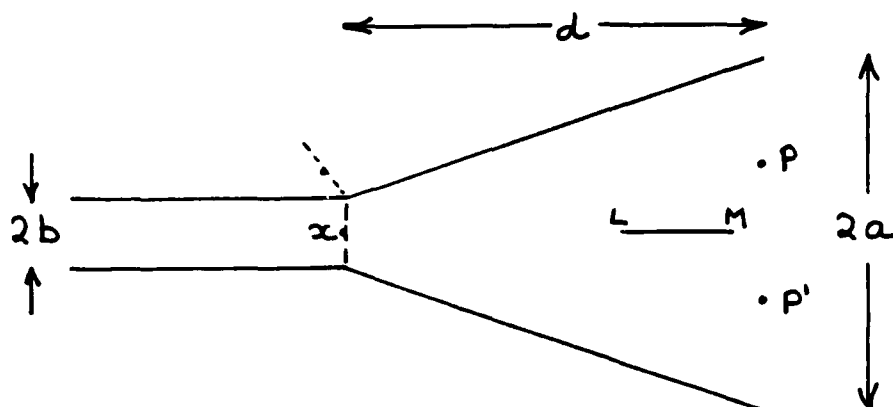


Figure 3    Horn aperture distributions  
for both the loaded and  
unloaded cases



**Figure 4.** Illustration of plate loaded H-plane sectorial horn.

Since the calculation was performed on a two dimensional sectorial horn the plate is represented as a line L-M in Figure 4. In the actual experiment it extended across the full width of the horn in the E-plane direction. In the calculation the length L-M was 28mm and the distance from point x to L (shown in Figure 4) was 70mm.

In addition to inverting phase, the plate acted as a diffraction limited aperture and blurred the radiation reflected from it.

To calculate this effect we analysed the situation with the plate removed and treated L-M as an empty aperture. Using L-M as a line of secondary sources we calculated the contribution that this has to the aperture distribution in the mouth of an unloaded horn. This involved a double integral, using equation 3 twice, initially over the neck of the horn, then over the line L-M, for each point in the mouth of the horn.

It was then assumed that when the line L-M is a reflector on both its upper and lower sides its contribution to the aperture distribution at the mouth of the horn is the same as that calculated for the empty line L-M but with the phase inverted. This should be a sufficiently good approximation to the required accuracy of the calculation.

A new aperture distribution across the mouth of the horn was calculated for the situation with the plate included, by subtracting twice the contribution from line L-M from the original empty horn distribution calculated in Section 3. This new aperture distribution is shown in Figure 3 and should be compared with the distribution for the empty horn also contained in Figure 3. The integrated intensity is virtually unchanged and the field still goes to zero at the edge of the horn but the new distribution has a peak intensity away



from the centre of the aperture which results in a sharper far field pattern.

## 5. CALCULATION OF THE FAR FIELD PATTERN FOR THE LOADED AND UNLOADED HORNS

To obtain the far field patterns we have calculated the Fourier transforms of the aperture distributions shown in Figure 3 for the loaded and unloaded horns. For the sake of completeness we have also included the obliquity factor  $\frac{1}{2} (1 + \cos \chi)$  used also in equation 3.

These far field patterns were calculated using the horn aperture distributions shown in Figure 3. These calculations were then repeated with the phases corrected to include the effect of lenses in the mouths of the horns. These lenses were designed to produce plane wavefronts on exit from their respective horns. The resulting computed far field patterns are shown in Figure 5 for the various lens-plate combinations.

Finally, the calculated pattern for the loaded lens-horn combination is compared in Figure 6 with an experimental measurement, carried out by Dr R Appleby and Mr C S Dickinson in RSRE.

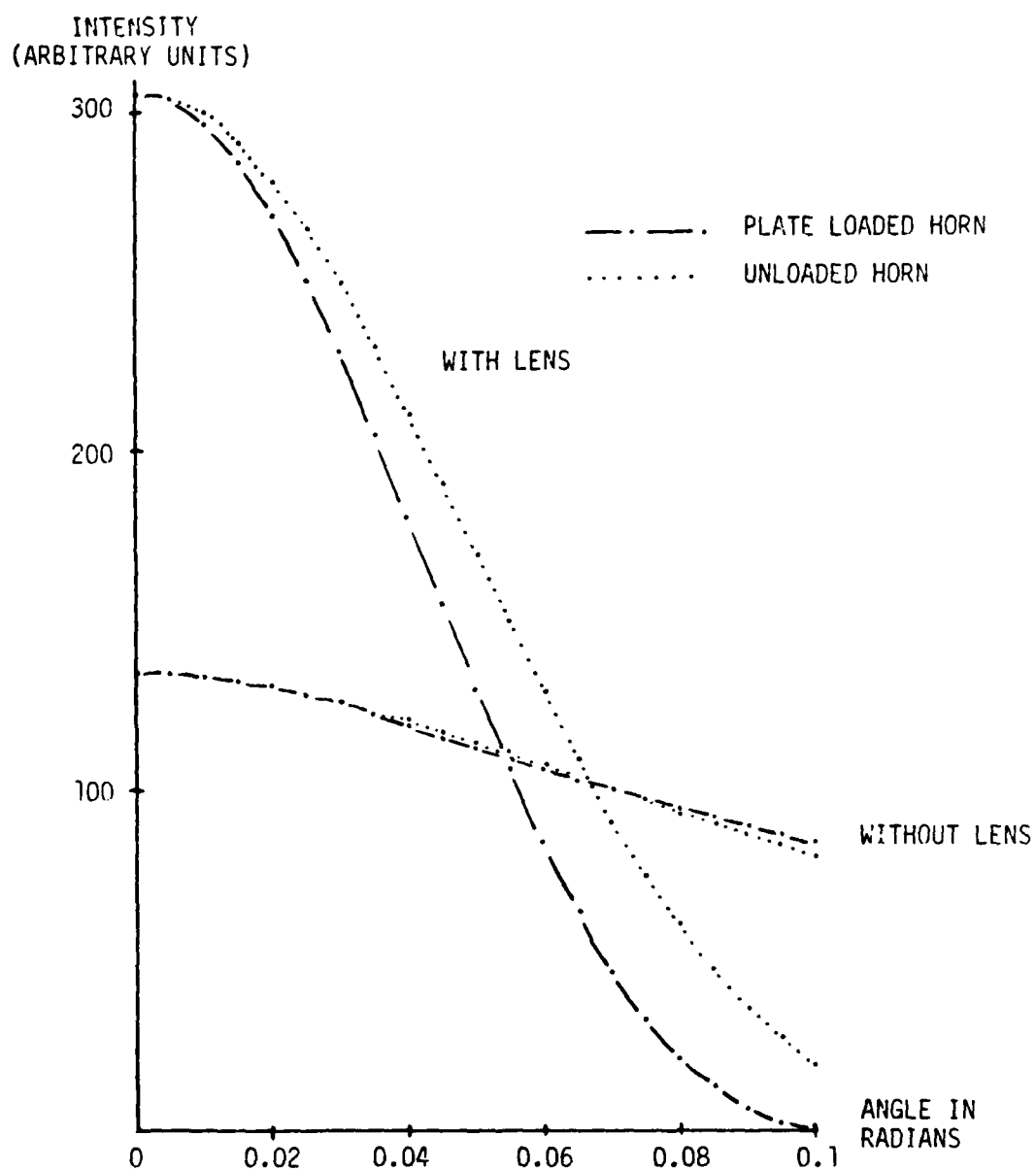
## 6. CONCLUSIONS

The experimental measurements and theoretical calculations are in excellent agreement and demonstrate that the inclusion of a plate within a horn may be used to sharpen the H-plane far field pattern. This is important for high resolution mmwave imaging.

The developed computer programs may now be used to optimise the dimensions of the loading of the horns in actual imaging systems.

## 7. REFERENCE

- [1] Born, M., and Wolf, E., Principles of optics; Pergamon Press, Oxford, 3rd Edition 1965.



**Figure 5** Calculated far field pattern for an unloaded horn and a plate loaded horn (dimensions given in Section 4), with and without phase correcting lenses

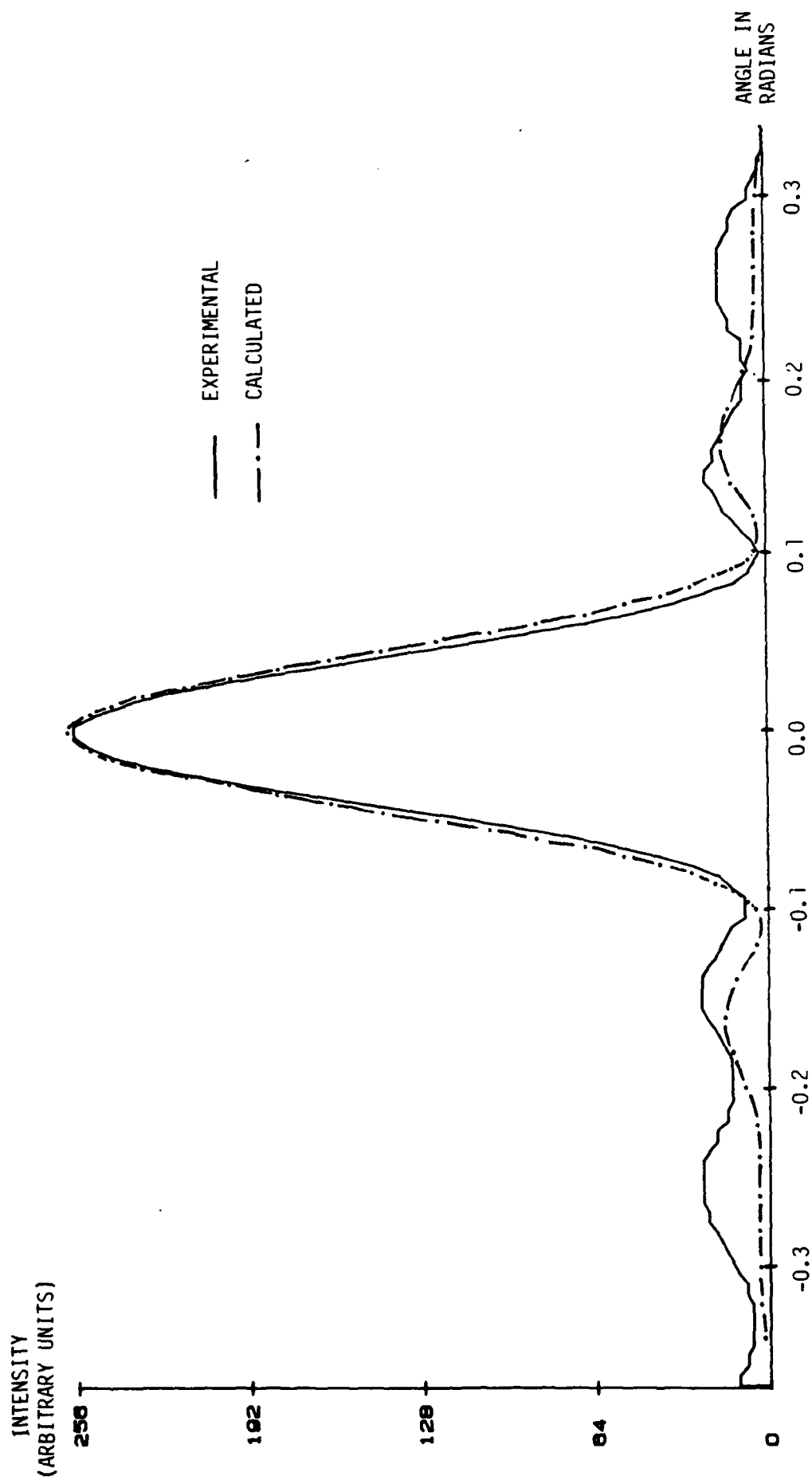


Figure 6 Calculated far field pattern for a plate loaded horn. The dimensions of the horn and plate are given in Section 4.

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